

# A new relationship between the stress multipliers of Garofalo equation for constitutive analysis of hot deformation in modified 9Cr–1Mo (P91) steel

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## ABSTRACT

Constitutive analysis is performed following the Garofalo sine–hyperbolic equation on the true stress–strain data obtained from isothermal hot compression tests on modified 9Cr–1Mo (P91) steel over a wide range of temperature (1123–1373 K) and strain rate (0.001–100 s<sup>-1</sup>). We propose a new relationship between the stress multipliers  $\alpha_g$  and  $\alpha_{ER}$  for P91 steel as  $(\alpha_g/\alpha_{ER}) = 0.27977 + 0.01531(n_p)^2$ ; where the adjustable stress multiplier  $\alpha_g$  is obtained by force fitting Garofalo equation ( $\dot{\epsilon} = \text{constant}[\sinh(\alpha_g\sigma)]^{n_g}$ ) and  $\alpha_{ER}$  is obtained as  $\alpha_{ER} = \beta_e/n_p$  from the data fitted to power ( $\dot{\epsilon} \propto \sigma^{n_p}$ ) and exponential ( $\dot{\epsilon} \propto \exp(\beta_e\sigma)$ ) laws for the entire stress/strain rate regime.  $\alpha_g$  was easily obtained knowing  $\alpha_{ER}$  and following this, the other constitutive parameters  $n_g$ ,  $Q$  and  $\ln A_g$  were determined and were found to be strain dependent except  $n_g$ . The successful prediction of flow stress has been supported by a higher correlation coefficient ( $R=0.994$ ) and a lower average absolute relative error (5.03%) for the entire investigated hot working domain.

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## 1. Introduction

True stress–true plastic strain data obtained from uniaxial hot compression tests are commonly employed for investigating the deformation behavior at elevated temperatures and high strain rates. The data obtained are described in terms of suitable constitutive equations that adequately correlate flow stress, strain, strain rate and temperature. These equations are of importance in modeling of hot working processes and the merit of these equations depends on how accurately they predict the flow stress under varying processing conditions. The developed constitutive equations have their limitations with regard to the optimum range of strain rates for calculating flow stresses; power–law which is preferred for creep [1–10] is suitable at low stresses/strain rates, while the exponential relationship [2–6] is valid at high stresses/strain rates. Though the exponential law was initially favored for hot working processes, but it has the inadequacy that it breaks down at higher temperatures and lower strain rates [2,10]. Ever since the proposition by Sellars and Tegart [11], and Jonas et al. [12], the Garofalo sine–hyperbolic equation [5] with Arrhenius term is well accepted for hot deformation at high strain rates. The sine–hyperbolic description of Zener–Hollomon parameter, i.e. temperature compensated strain rate has been widely employed for predicting flow stress. The constitutive analysis in hot working

for austenitic stainless steels, carbon and alloy steels, ferritic steels and Al alloys has been comprehensively reviewed by McQueen and Ryan [2]. Recently, it is critically reviewed by Lin and Chen [13].

The power–law creep behavior has been reviewed exhaustively by Kassner and Perez-Prado [3,4]. For high temperature creep, power–law relation between steady-state creep rate ( $\dot{\epsilon}_s$ ) and stress ( $\sigma$ ) with Arrhenius term for temperature ( $T$ ) dependence (i.e.  $\dot{\epsilon}_s = A/\sigma^n e^{-Q/RT}$ ) is valid at lower/intermediate stresses, but it breaks down at higher stresses. Similarly, the exponential law (i.e.  $\dot{\epsilon}_s = A^{II} \exp(\beta\sigma) e^{-Q/RT}$ ) that is obeyed at higher stresses gradually loses its linearity at lower stresses. The Garofalo [5] sine–hyperbolic expression (i.e.  $\dot{\epsilon}_s = A[\sinh(\alpha\sigma)]^n e^{-Q/RT}$ ) suitably correlates stress dependence for the entire stress regime, as it reduces to power–law at low stresses (valid for  $\alpha\sigma < 0.8$ ) and exponential at higher stress limits (applicable for  $\alpha\sigma > 1.2$ ). In these correlations,  $R$  is the universal gas constant,  $Q$  is the activation energy term,  $A$ ,  $A^I$ ,  $A^{II}$  are constants, and it follows that  $A^I = A(\alpha)^n$ ,  $A^{II} = A/2^n$  [5]. During creep, as strain rate is a derived quantity, the power–law stress exponent  $n$  at lower stresses and similarly  $\beta$  in the high stress region are distinctly known which allows clearly the determination of  $\alpha$  (i.e.  $\alpha = \beta/n$ ) in Garofalo equation that suitably describes stress dependence of creep rate for the entire stress regime.

On the other hand, the situation during hot working is that strain rate is imposed and flow stress data is force fitted by plotting  $\ln(\dot{\epsilon})$  vs.  $\ln[\sinh(\alpha\sigma)]$  and the adjustable stress multiplier  $\alpha$  (MPa<sup>-1</sup>) is iterated that brings  $(\alpha\sigma)$  in to the correct range yielding parallel linear lines (with slope as  $n$ ) for different temperatures [2]. The

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Garofalo sine–hyperbolic equation for stress dependence of strain rate ( $\dot{\epsilon}$ ) with Arrhenius term for hot working range can be expressed as

$$\dot{\epsilon} = A_g [\sinh(\alpha_g \sigma)]^{n_g} \exp(-Q/RT), \quad (1)$$

where  $Q$  is the apparent activation energy,  $A_g$ ,  $n_g$  and  $\alpha_g$  are constants, and this relation is valid for the entire strain rate/stress regime. As mentioned before, stress exponent  $n_g$  and Garofalo stress multiplier  $\alpha_g$  ( $\text{MPa}^{-1}$ ) are determined by force fitting Eq. (1) for the flow stress data. Eq. (1) is generally used for fitting the peak or steady-state flow stress. But, recent studies [14–20] show that it can be employed for predicting flow stress at different strains with an objective to develop a strain dependent constitutive equation. The power–law and exponential relations for hot deformation are given as

$$\dot{\epsilon} = A_p(\sigma)^{n_p} \exp(-Q/RT), \quad (2)$$

$$\dot{\epsilon} = A_e \exp(\beta_e \sigma) \exp(-Q/RT), \quad (3)$$

where  $n_p$ ,  $\beta_e$ , are stress exponents;  $A_p$  and  $A_e$  are constants.

Lin et al. [13,15,16] and Mandal et al. [17] have fitted power–law and exponential relations for the data obtained by hot compression tests. Their results show that the fit is for the entire strain rate (or stress) range and in contrast to creep, no deviation is revealed in these plots. Hence, we designate  $n_p$  determined following Eq. (2) as power–law stress exponent for the entire stress/strain rate range and can be obtained as reciprocal of slope of lines in the plot of  $\ln(\sigma)$  vs.  $\ln(\dot{\epsilon})$  at different temperatures for a given strain, and is given as

$$n_p = \left. \frac{\partial \ln(\dot{\epsilon})}{\partial \ln(\sigma)} \right|_T. \quad (4)$$

Similarly,  $\beta_e$  in Eq. (3) corresponds to reciprocal of slope of straight lines in the plot of  $\sigma$  vs.  $\ln(\dot{\epsilon})$  at a given strain for the entire strain rate regime and is given as

$$\beta_e = \left. \frac{\partial \ln(\dot{\epsilon})}{\partial \sigma} \right|_T. \quad (5)$$

Then the stress multiplier  $\alpha$  ( $\text{MPa}^{-1}$ ) can be determined [13,15–17] as  $\alpha = \beta_e/n_p$  and was used for predicting the flow stress at different strains. In this work, this  $\alpha$  is designated as  $\alpha_{ER}$  (i.e.  $\alpha_{ER} = \beta_e/n_p$ ), where the subscript ER indicates that it is determined by fitting power and exponential laws for the entire strain rate range. Whereas, those determined by force fitting Garofalo equation (1) are designated as  $\alpha_g$  and  $n_g$ .

In the present study, we address an important question whether any relationship exists between the two stress multipliers, viz.  $\alpha_g$  and  $\alpha_{ER}$ . This is useful, because evaluating the adjustable  $\alpha_g$  by iteration method as described earlier involves much of computational time. And, for a given material, once a relationship between  $\alpha_g$  and  $\alpha_{ER}$  is established for the investigated strain rate–temperature domain,  $\alpha_g$  can be evaluated by knowing  $\alpha_{ER}$  which is much easier to be determined. In this paper, we propose a new relationship between  $\alpha_g$  and  $\alpha_{ER}$  in terms of  $n_p$  for the data obtained from hot compression tests on modified 9Cr–1Mo (P91) steel over a wide range of strain rate and temperature. Using the  $\alpha_g$  values calculated following the proposed relationship; the other constitutive parameters  $n_g$ ,  $Q$  and  $\ln A_g$  were determined at various strains. On incorporating the strain dependence of these parameters, prediction of flow stress was performed following the Garofalo equation. The predicted flow stresses were compared with those experimentally obtained over the entire investigated strain rate and temperature domain.

## 2. Experimental

True stress–true plastic strain data obtained (in the strain range 0.1–0.5, at steps of 0.05) from isothermal hot compression tests on modified 9Cr–1Mo (P91) steel [18,19] over a wide range of temperatures (1123–1373 K at intervals of 50 K) and strain rates (0.001, 0.01, 0.1, 1, 10 and 100  $\text{s}^{-1}$ ) is employed for the constitutive analysis. The steel was in normalized (1323 K for 1 h) and tempered (1033 K for 3 h) condition. The chemical composition of the steel (in wt%) is as follows: 0.102C, 0.374Mn, 8.946Cr, 0.914Mo, 0.227Si, 0.064Cu, 0.107Ni, 0.182V, 0.03Al, 0.075Nb, 0.003Ti, 0.0594N, and balance Fe. The specimens with 10 mm diameter and 15 mm height, machined with a very good surface finish of the order of  $\nabla\nabla\nabla$  (N6 grade) were used for compression tests so as to exclude any surface effects on flow behavior. A small hole of 0.8 mm diameter and 5 mm depth was drilled at mid height of the specimen to insert a thermocouple, which was used to measure the actual temperature of the specimen. Isothermal hot compression tests were conducted using a computer controlled servo–hydraulic testing machine (DARTEC, Stourbridge, UK) with a maximum load capacity of 100 kN. The machine is equipped with a control system to impose exponential decay of the actuator speed to obtain constant true strain rates. A resistance heating split furnace with SiC heating elements was used to surround the platens and specimen. The specimens were coated with a borosilicate glass paste that acted as a lubricant as well as protective coating. A Nicolet transient recorder was used to record the adiabatic temperature rise during hot deformation. Standard equations were used to convert the load–stroke data to true stress–true strain data. The elastic region was subtracted from the true stress–strain curve [21] to get true stress–true plastic strain in the range 0.1–0.5. Flow stress data obtained at different processing conditions were corrected for adiabatic temperature rise, if any, by linear interpolation (point to point) between  $\ln \sigma$  and  $1/T$ , where  $T$  is the absolute test temperature. The appearance of the specimens before and after testing showed no significant barreling upon testing [18].

## 3. Results and discussion

### 3.1. Analysis of flow stress data to determine stress multiplier $\alpha_{ER}$

The flow stress data obtained at different strains (0.1–0.5 at steps of 0.05) were analyzed to determine  $\alpha_{ER}$ . Towards this,  $n_p$  and  $\beta_e$  need to be evaluated and this is described in the following, at a true strain = 0.15 as an example. The values of  $n_p$  and  $\beta_e$  were determined as reciprocal of slopes of lines in the plots  $\ln(\sigma)$  vs.  $\ln(\dot{\epsilon})$  and  $\sigma$  vs.  $\ln(\dot{\epsilon})$ , respectively at different temperatures (Fig. 1a and b). The mean value of  $n_p$  and  $\beta_e$  were found to be 8.3984 and 0.053459  $\text{MPa}^{-1}$ , respectively at strain = 0.15. For calculating  $\alpha_{ER}$ , the respective values of  $n_p$  and  $\beta_e$  obtained at different temperatures (1123–1373 K, at intervals of 50 K) were used.  $\alpha_{ER}$  at each temperature was obtained as  $\alpha_{ER} = \beta_e/n_p$  and then the average value of  $\alpha_{ER}$  was obtained as 0.00676  $\text{MPa}^{-1}$  at a strain of 0.15. This procedure was repeated at different strains for the strain range (0.1–0.5) at steps of 0.05 to determine  $n_p$ ,  $\beta_e$  and  $\alpha_{ER}$ . The strain dependence of the average values of  $\alpha_{ER}$  and  $n_p$  is shown in Fig. 2. In our recent study [18], the adjustable stress multiplier  $\alpha_g$  (and  $n_g$ ) were determined by force fitting Garofalo equation (i.e. Eq. (1)) for the flow stress data on the P91 steel for the strain range 0.1–0.5. It may be noted (Fig. 3 in Ref. [18]) that though  $\alpha_g$  values decreased from 0.00889 to 0.00751 with increasing strain (0.1–0.5),  $n_g$  did not show any perceptible variation with strain (i.e. almost was constant) and the average value was obtained as 5.2248.

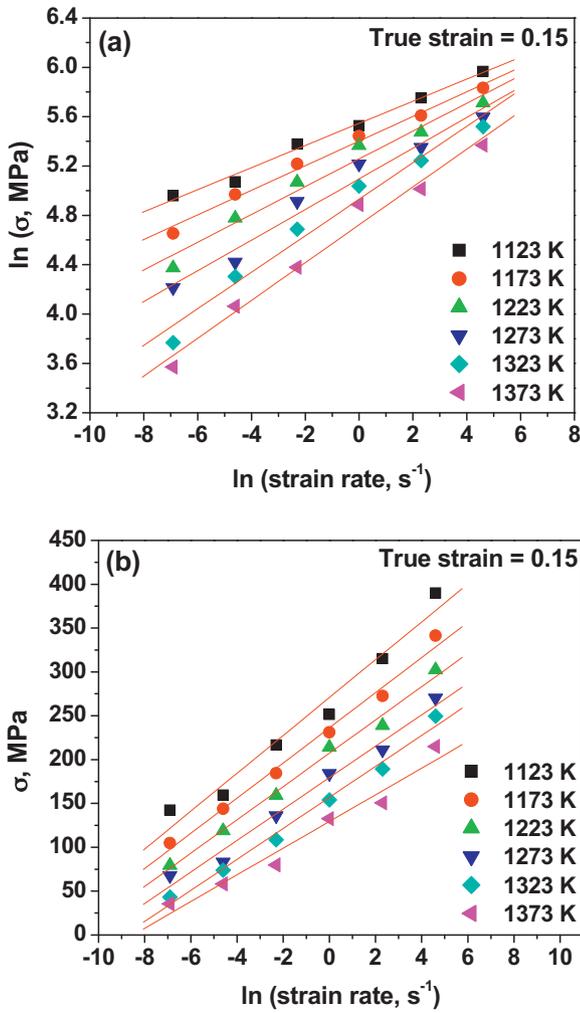


Fig. 1. Typical plots showing the determination of (a)  $n_p$  from  $\ln \sigma$  vs.  $\ln \dot{\epsilon}$  and (b)  $\beta_e$  from  $\sigma$  vs.  $\ln \dot{\epsilon}$  at constant strain = 0.15.

3.2. Relationship between the stress multipliers  $\alpha_g$  and  $\alpha_{ER}$

The important observations that led us to the relationship between  $\alpha_g$  and  $\alpha_{ER}$  for P91 steel are presented below (for strain = 0.15 as an example). In Section 3.1, it was observed that the average value of  $n_p = 8.3984$  at strain = 0.15 (Fig. 1a). However (at strain = 0.15), it was noticed that  $n_p$  was found to decrease from 11.058 to 6.388 with increasing temperature from 1123 to 1373 K. The analysis also revealed that the same trend was observed for the variation of  $(\alpha_g/\beta_e)$  with temperature, i.e.  $(\alpha_g/\beta_e)$  was found

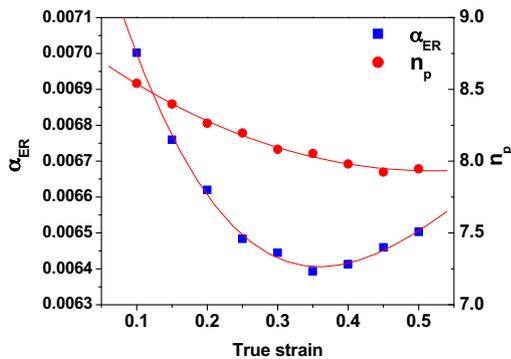


Fig. 2. Variation of  $\alpha_{ER}$  and  $n_p$  with true strain.

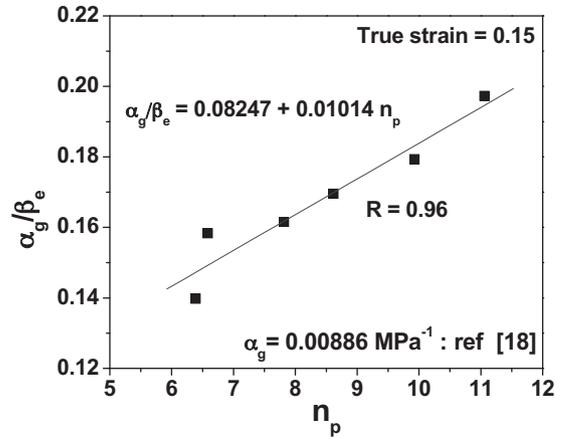


Fig. 3. Linear dependence between  $(\alpha_g/\beta_e)$  and  $n_p$  at strain = 0.15.

to decrease from 0.1973 to 0.1397 with increase in temperature from 1123 to 1373 K;  $\alpha_g = 0.00886 \text{ MPa}^{-1}$  was taken from our previous work [18] at strain = 0.15. These observations prompted us to believe that  $(\alpha_g/\beta_e)$  would increase with increase in  $n_p$  and such a plot is shown in Fig. 3, typically for strain = 0.15. It could be seen that the data could be fitted by a straight line suggesting that  $(\alpha_g/\beta_e) \propto n_p$ . Similar behavior was noticed at all strains for the strain range 0.1–0.5 at steps of 0.05.

From the definition of  $\alpha_{ER} = \beta_e/n_p$ , it follows that  $(\alpha_g/\beta_e) = \alpha_g/(\alpha_{ER} \times n_p) \propto n_p$ . Therefore,  $(\alpha_g/\alpha_{ER})$  would show a linear dependence with  $(n_p)^2$ . The  $(\alpha_g/\alpha_{ER})$  was calculated at various strains (0.1–0.5, steps of 0.05) for different temperatures (1123–1373 K, 50 K interval). The plot of  $(\alpha_g/\alpha_{ER})$  vs.  $(n_p)^2$  is shown in Fig. 4 which clearly illustrates that the data could be described by linear relationship. Hence, the new relationship between the stress multipliers  $\alpha_g$  and  $\alpha_{ER}$  for the modified 9Cr–1Mo (P91) steel can be expressed as

$$\frac{\alpha_g}{\alpha_{ER}} = 0.27977 + 0.01531(n_p)^2 \tag{6}$$

The above relationship can be rewritten as

$$\frac{\alpha_g^\epsilon}{\alpha_{ER}^\epsilon} = 0.27977 + 0.01531(n_p^\epsilon)^2, \tag{7}$$

where the superscript  $\epsilon$  indicates the strain dependence. An important implication of the above proposed relationship (Eq. (7)) is that knowing  $\alpha_{ER}$  and  $n_p$  at each strain (Fig. 2),  $\alpha_g$  values can be easily determined. For strain = 0.15, knowing  $\alpha_{ER} = 0.00676$  and

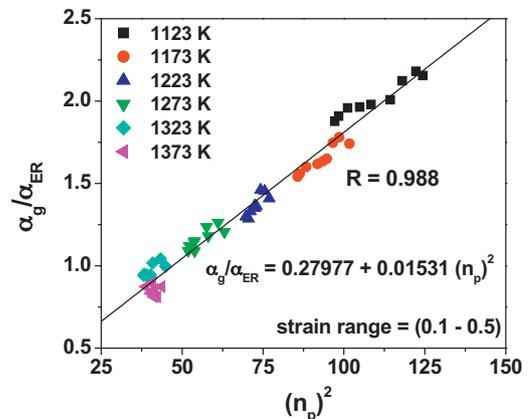


Fig. 4. Relationship between  $\alpha_g$  and  $\alpha_{ER}$  in terms of  $n_p$  for the strain range (0.1–0.5) at intervals of 0.05.

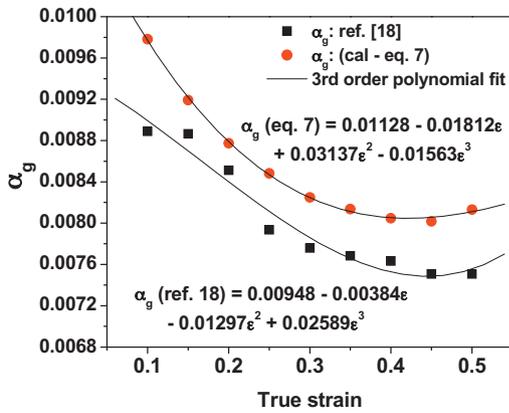


Fig. 5. Variation of  $\alpha_g$  with true strain.  $\alpha_g$  values obtained in our previous work [18] are also shown along with the polynomial fits.

$n_p = 8.3984$ ,  $\alpha_g$  was calculated as 0.0091912 according to Eq. (7) and the same was obtained at different strains. Fig. 5 shows the variation of  $\alpha_g$  with strain and for comparison, the earlier reported  $\alpha_g$  values [18] are also shown in this plot. It can be seen that  $\alpha_g$  ( $\text{MPa}^{-1}$ ) values obtained in this study decreased from 0.00978 to 0.00802 with increasing strain and this is in correspondence with those observed earlier based on the iteration method. Also, the  $\alpha_g$  values obtained following Eq. (7) were found to be described best by the polynomial of the 3rd order and the coefficients of the polynomial are given in Fig. 5. After calculating  $\alpha_g$ , following the Garofalo sine–hyperbolic Eq. (1), the other constitutive parameters such as  $n_g$ ,  $Q$  and  $\ln A_g$  were evaluated and this is given in the next section.

### 3.3. Evaluation of other parameters in Garofalo equation and their strain dependence

Using the calculated value of  $\alpha_g$ , following Eq. (1), the plot of  $\ln[\sinh(\alpha_g \sigma)]$  vs.  $\ln(\dot{\epsilon})$  could be drawn for a given strain at different temperatures that would yield parallel lines and reciprocal of the slope of these lines gives  $n_g$ . Such a typical plot at strain = 0.15 is shown in Fig. 6 and the average value of  $n_g$  was obtained as 5.0021. This procedure was repeated at various strains (0.1–0.5, at steps of 0.05) to calculate  $n_g$  at different strains. The variation of average values of  $n_g$  with strain is shown in Fig. 7 and those reported previously [18] are also given for comparison. It can be seen that  $n_g$  did not show any noticeable variation with strain, and the average of values obtained at different strains was found to be 5.0369. This is

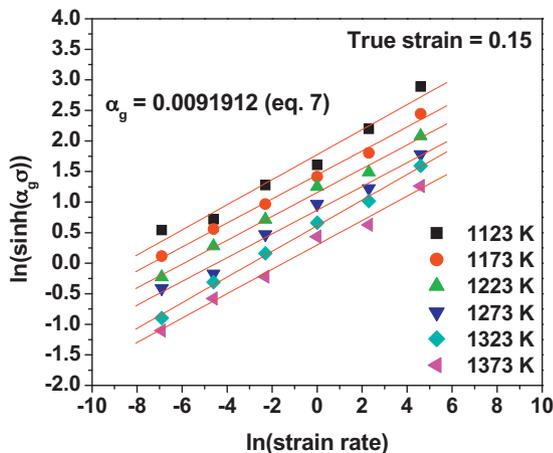


Fig. 6. Typical plot of  $\ln[\sinh(\alpha_g \sigma)]$  vs.  $\ln(\dot{\epsilon})$  at strain = 0.15 for evaluation of  $n_g$ .

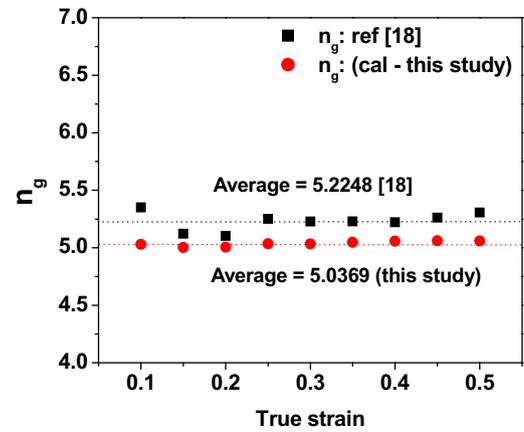


Fig. 7. Plot of  $n_g$  with true strain. The  $n_g$  values obtained in our previous work [18] are also shown.

in almost accordance with the earlier reported value of  $n_g = 5.2248$  [18].

Knowing  $n_g$  and  $\alpha_g$ , the other parameters  $Q$  and  $\ln A_g$  in Eq. (1) can be determined. Eq. (1) on rearrangement can be written as

$$n_g \ln(\sinh(\alpha_g \sigma)) = \ln \dot{\epsilon} + \frac{Q}{RT} - \ln A_g \quad (8)$$

For a particular strain, the intercepts obtained in the plot of  $n_g \ln(\sinh(\alpha_g \sigma))$  vs.  $\ln(\dot{\epsilon})$  was plotted against  $1/T$ . In this plot,  $Q$  was obtained from the slope (slope =  $Q/R$ ), while the intercept gives  $\ln A_g$ . A similar procedure was repeated to find  $Q$  and  $\ln A_g$  at various strains. Fig. 8 shows the strain dependence of  $Q$  and  $\ln A_g$ . It is seen that the range of values of  $Q$  (368.7–395.9 kJ/mol) and  $\ln A_g$  (30.7–33.7) obtained in this study showed not only the similar trend but were also in correspondence with those reported in our previous study [18]. Further,  $Q$  and  $\ln A_g$  were found to be best described by the 3rd order polynomial fit with very good correlation and generalization, and the coefficients of the polynomials are given in the figure.

### 3.4. Prediction of flow stress incorporating compensation for strain

The general assumption is that the influence of strain on elevated temperature flow behavior is insignificant and thereby it is not considered in Garofalo equation (1). However, it is seen that

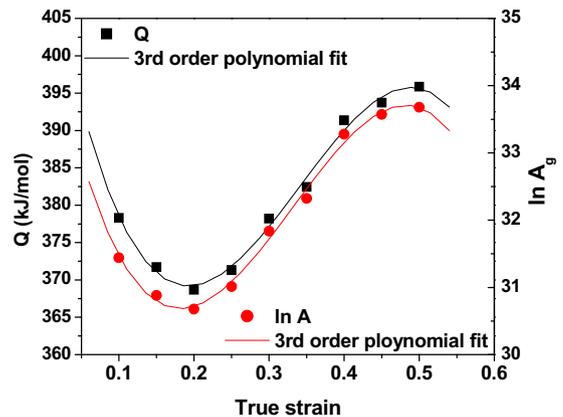
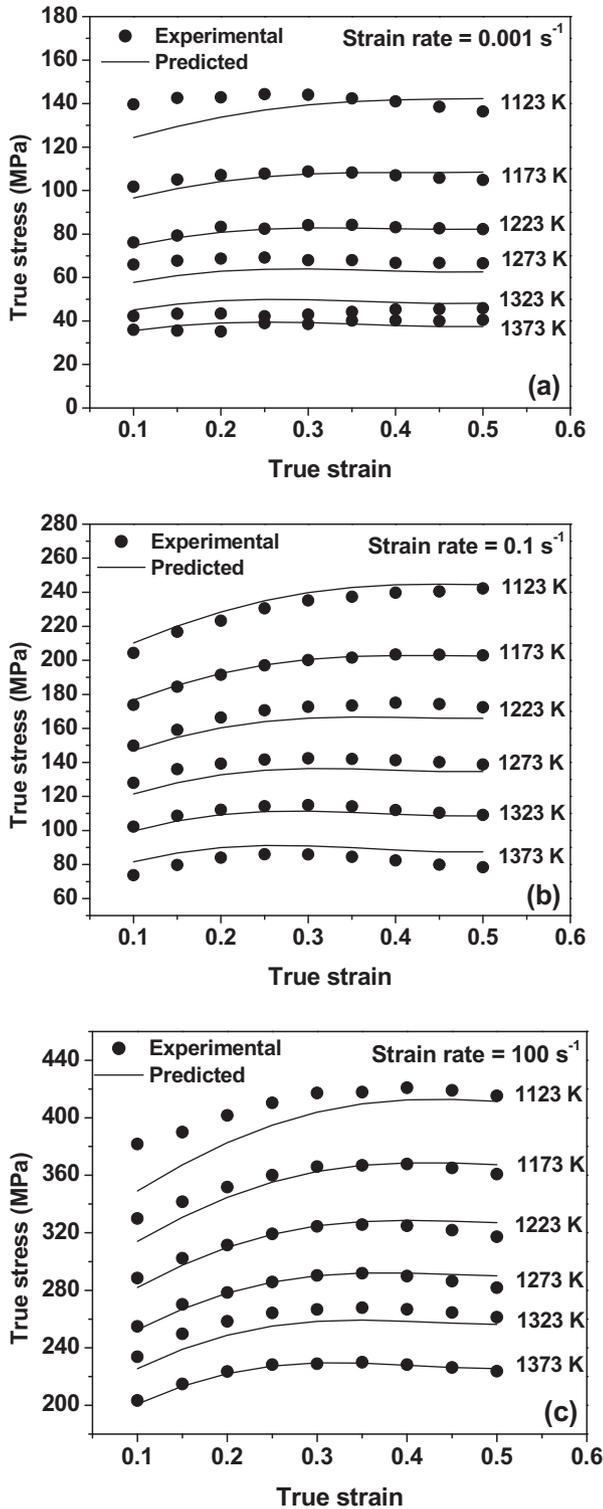
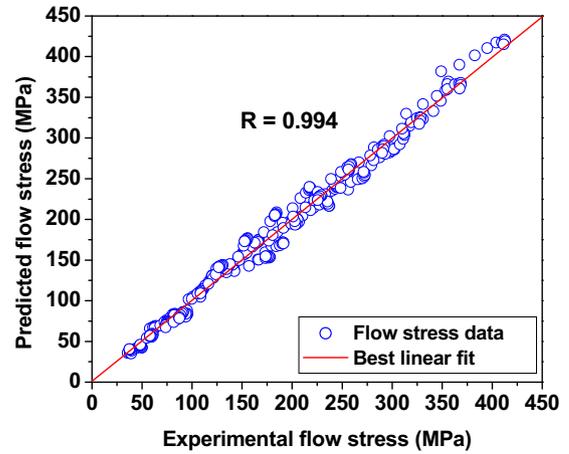


Fig. 8. Variation of  $Q$  and  $\ln A_g$  with true strain for the strain range (0.1–0.5) at intervals of 0.05. The polynomials for  $Q$  and  $\ln A_g$  are given as  $Q = 417.36 - 576.57 \epsilon + 2087.98 \epsilon^2 - 2043.22 \epsilon^3$  and  $\ln A_g = 35.34 - 58.60 \epsilon + 222.018 \epsilon^2 - 222.92 \epsilon^3$ .



**Fig. 9.** Typical predicted flow curves and its comparison with those experimentally obtained at strain rates of (a)  $0.001 \text{ s}^{-1}$ , (b)  $0.1 \text{ s}^{-1}$  and (c)  $100 \text{ s}^{-1}$  in the temperature range 1123–1373 K at intervals of 50 K.

$\alpha_g$  (Fig. 5),  $Q$  and  $\ln A_g$  (Fig. 8) showed significant variation with strain, whereas  $n_g$  was almost constant with  $n_g = 5.0369$  (Fig. 7). Hence the compensation for strain (i.e. strain dependence of constitutive parameters  $\alpha_g$ ,  $Q$  and  $\ln A_g$ ) needs to be incorporated in to Eq. (1) while predicting flow stress. It is already mentioned that these parameters were best described by the polynomial functions of 3rd order with very good correlation and generalization. Using



**Fig. 10.** Correlation between the predicted and experimental flow stresses for the strain range 0.1–0.5 (at intervals of 0.05) over the entire strain rate ( $0.001$ – $100 \text{ s}^{-1}$ ) and temperature range (1123–1373 K).

these respective polynomials,  $\alpha_g$ ,  $Q$  and  $\ln A_g$  can be evaluated for a given strain. Once these parameters are known, the flow stress at a specified strain for a given strain rate and temperature can be predicted following the equation below which is obtained by rearranging Eq. (1) as

$$\sigma = \frac{1}{\alpha_g} \left( \sinh^{-1} \left( \frac{Z}{A_g} \right)^{\frac{1}{n_g}} \right), \quad (9)$$

where  $Z$  is the well known Zener–Hollomon parameter given by

$$Z = \dot{\epsilon} \exp(Q/RT) \quad (10)$$

Prediction of flow stress was carried out following Eq. (9) for the investigated strain rate ( $0.001$ – $100 \text{ s}^{-1}$ ) and temperature range (1123–1373 K at intervals of 50 K). The typical predicted flow curves and their comparison with those experimentally obtained are shown in Fig. 9(a)–(c) for the data at lower ( $0.001 \text{ s}^{-1}$ ), intermediate ( $0.1 \text{ s}^{-1}$ ) and higher ( $100 \text{ s}^{-1}$ ) strain rates at various temperatures. It could be observed that the predicted flow stresses are in good agreement with those experientially measured. Almost similar prediction accuracy was also noticed at other strain rates and therefore is not emphasized.

To quantify the predictability of flow stress, the standard statistical parameters such as correlation coefficient ( $R$ ) and average absolute relative error ( $\Delta$ ) were also determined for the data at various strains (0.1–0.5) at intervals of 0.05 for the entire strain rate–temperature regime according to expressions given below

$$R = \frac{\sum_{i=1}^{i=N} (\sigma_{\text{exp}}^i - \bar{\sigma}_{\text{exp}})(\sigma_p^i - \bar{\sigma}_p)}{\sqrt{\sum_{i=1}^{i=N} (\sigma_{\text{exp}}^i - \bar{\sigma}_{\text{exp}})^2 \sum_{i=1}^{i=N} (\sigma_p^i - \bar{\sigma}_p)^2}}, \quad (11)$$

$$\Delta = \frac{1}{N} \sum_{i=1}^{i=N} \left| \frac{\sigma_{\text{exp}}^i - \sigma_p^i}{\sigma_{\text{exp}}^i} \right| \times 100, \quad (12)$$

where  $\sigma_{\text{exp}}$  is the experimental flow stress,  $\sigma_p$  is the predicted flow stress,  $\bar{\sigma}_{\text{exp}}$  and  $\bar{\sigma}_p$  are the mean values of  $\sigma_{\text{exp}}$  and  $\sigma_p$ , respectively, and  $N$  is the total number of data points. The correlation coefficient provides information on the strength of linear relationship between the observed and the computed values, whereas  $\Delta$  is computed through a term by term comparison of the relative error. It can be seen from Fig. 10 that a good correlation ( $R = 0.994$ ) is obtained between the experimental and the predicted data. The average absolute relative error was found to be 5.03% for the entire range of strain, strain rate and temperature. These statistical analyses reveal the good predictability of the Garofalo

equation for strain rates ( $0.001\text{--}100\text{ s}^{-1}$ ) in the temperature range ( $1123\text{--}1373\text{ K}$ ). Most importantly, it emphasizes that  $\alpha_g$  and in turn the other constitutive parameters  $n_g$ ,  $Q$  and  $\ln A_g$  obtained based on the proposed new relationship between the stress multipliers  $\alpha_g$  and  $\alpha_{ER}$  (i.e. Eq. (7)) can be successfully employed for predicting the flow behavior of modified 9Cr–1Mo (P91) steel in the investigated strain rate and temperature domain. The relationship between  $\alpha_g$  and  $\alpha_{ER}$  needs to be explored for other materials and this is being pursued on compression tests data for a 15Cr–15Ni Ti modified austenitic stainless steel (alloy D9).

#### 4. Conclusions

The constitutive analysis was performed on the flow stress data obtained from isothermal hot compression tests on modified 9Cr–1Mo (P91) steel over a wide range of temperature ( $1123\text{--}1373\text{ K}$ ), strain rate ( $0.001\text{--}100\text{ s}^{-1}$ ) and strain ( $0.1\text{--}0.5$ ). The principal conclusions from this study are given below.

A new relationship is proposed between the stress multipliers  $\alpha_g$  and  $\alpha_{ER}$  for P91 steel as  $(\alpha_g/\alpha_{ER}) = 0.27977 + 0.01531(n_p)^2$ ; where the adjustable stress multiplier  $\alpha_g$  is obtained by force fitting the Garofalo sine–hyperbolic equation ( $\dot{\epsilon} = \text{constant}[\sinh(\alpha_g\sigma)]^{n_g}$ ) and  $\alpha_{ER}$  is obtained as  $\alpha_{ER} = \beta_e/n_p$  from the data fitted to power ( $\dot{\epsilon} \propto \sigma^{n_p}$ ) and exponential ( $\dot{\epsilon} \propto \exp(\beta_e\sigma)$ ) laws for the entire stress/strain rate regime.

Following the relationship between the stress multipliers,  $\alpha_g$  was easily obtained knowing  $\alpha_{ER}$  and using this calculated value of  $\alpha_g$ , the other constitutive parameters  $n_g$ ,  $Q$  and  $\ln A_g$  were determined. While  $n_g$  was found to be independent of strain,  $\alpha_g$ ,  $Q$  and  $\ln A_g$  showed significant variation with strain, and were described by a 3rd order polynomial with very good correlation and generalization.  $Q$  was in the range  $368.7\text{--}395.9\text{ kJ/mol}$  which corresponds to the activation energy reported for hot working of ferritic steels.

The Garofalo equation (incorporating strain compensation) was found to predict the flow stress very well and this was reflected by a higher correlation coefficient ( $R = 0.994$ ) and a lower average abso-

lute relative error (5.03%) for the entire investigated hot working domain.

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#### References

- [1] J.E. Bird, A.K. Mukherjee, J.E. Dorn, in: D.G. Brandon, A. Rosen (Eds.), *Quantitative Relation Between Properties and Microstructures*, Israel University Press, Jerusalem, 1969, pp. 255–342.
- [2] H.J. McQueen, N.D. Ryan, *Mater. Sci. Eng. A* 322 (2002) 43–63.
- [3] M.E. Kassner, M.-T. Perez-Prado, *Prog. Mater. Sci.* 45 (2000) 1–102.
- [4] M.E. Kassner, M.-T. Perez-Prado, *Fundamentals of Creep in Metals and Alloys*, 2nd edition, Elsevier, Amsterdam, 2008.
- [5] F. Garofalo, *Fundamentals of Creep and Creep–Rupture in Metals*, Mc Millan, New York, 1965, pp. 50–54.
- [6] J. Cadek, *Creep in Metallic Materials*, Elsevier, Amsterdam, 1988, pp. 52–64.
- [7] R.W. Evans, B. Wilshire, *Introduction to Creep*, The Inst. of Materials, London, 1993.
- [8] A.K. Mukherjee, *Mater. Sci. Eng. A* 322 (2002) 1–22.
- [9] C. Phaniraj, *Creep Behaviour of AISI 304 Stainless Steel – Analysis and Implications of First Order Kinetics*, Ph.D. thesis, Indian Institute of Technology, Bombay, 1997.
- [10] F.A. Slooff, J. Zhou, J. Duszczkyk, L. Katgerman, *Scripta Mater.* 57 (2007) 759–762.
- [11] C.M. Sellars, W.J. McG. Tegart, *Acta Metall.* 14 (1966) 1136–1138.
- [12] J. Jonas, C.M. Sellars, W.J. McG. Tegart, *Metall. Rev.* 14 (1969) 1–24.
- [13] Y.C. Lin, X.-M. Chen, *Mater. Des.* 32 (2011) 1733–1759.
- [14] F.A. Slooff, J. Zhou, J. Duszczkyk, L. Katgerman, *J. Mater. Sci.* 43 (2008) 7165–7170.
- [15] Y.C. Lin, M.-S. Chen, J. Zhong, *Comput. Mater. Sci.* 42 (2008) 470–477.
- [16] Y.C. Lin, Y.-C. Xia, X.-M. Chen, M.-S. Chen, *Comput. Mater. Sci.* 50 (2010) 227–233.
- [17] S. Mandal, V. Rakesh, P.V. Sivaprasad, S. Venugopal, K.V. Kasiviswanathan, *Mater. Sci. Eng. A* 500 (2009) 114–121.
- [18] D. Samantaray, S. Mandal, A.K. Bhaduri, *Mater. Des.* 31 (2010) 981–984.
- [19] D. Samantaray, C. Phaniraj, S. Mandal, A.K. Bhaduri, *Mater. Sci. Eng. A* 528 (2011) 1071–1077.
- [20] J. Castellanos, I. Rieiro, M. Carsi, O.A. Ruano, *J. Mater. Sci.* 45 (2010) 5522–5527.
- [21] D. Samantaray, S. Mandal, A.K. Bhaduri, *Mater. Des.* 32 (2011) 2797–2802.